Journal of Applied Mechanics and Technical Physics, Vol. 38, No. 2, 1997

DISSIPATION OF DYNAMIC-LOADING ENERGY IN QUASI-ELASTIC DEFORMATION PROCESSES IN ROCKS

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UDC 624.131+539.215

It is believed [1, 2] that at sufficiently low loading levels, rocks behave as elastic solids, which are capable of dissipating deformation energy only at pressures that lead to the occurrence of an irreversible deformation component due to transition to a plastic state or due to damage. It is shown below that in dynamic loading, irreversible energy absorption is also observed under loads that are significantly lower than the elastic limit, and this can be associated with both viscosity and changes in the internal structure of rocks.

The general mechanism of variation in the energy capacity of dynamic deformation with variation in loading intensity and conditions is established on the basis of experiments with various intrusive rocks (granites, granodiorites, granite-gneiss, etc.). Qualitative data on the dissipation of dynamic-loading energy in the regions of quasi-elastic and elastoplastic deformation of intrusive rocks are compared.

Rocks are complex deformable media. This complexity is due to the features of their physical state (polymineral composition, heterogeneity, multiphase and granular structure, porosity, structural strength, etc.), which affect the deformation process from the very beginning of loading and which are reflected on the shape of strain diagrams. Since the overwhelming majority of rocks undergo compression loads, the discussion below is concerned with this type of dynamic load. At the beginning of loading, partial closure of pore space (microdefects) [3] has an effect on the stress–strain relationship, causing nonlinearity of strain diagrams in the region of low pressures. Under sufficiently large loads, the strain of the mineral rock material exceeds considerably the strain of the pore space, and the behavior of the rock becomes nearly linear-elastic. As microdefects are accumulated or the plastic-strain component increases, a departure from this behavior is observed at stresses comparable with the ultimate or yield stresses.

Figure 1 shows $\sigma(\varepsilon)$ diagrams for uniaxial dynamic (a) and static (b) compression. The diagrams are plotted on the basis of experimental studies of rocks of various origins performed on a measuring complex [4]. Curve numbers 1–6 correspond to the ordinal numbers of rocks from some deposits of refractory and nonmetalliferous raw materials and also of the natural gas of Ukraine in Table 1, which gives brief data on their physicomechanical properties: density ρ, porosity n, longitudinal elastic wave velocities v_{long}, Poisson's ratio ν, and strength in uniaxial static compression strength σ_0.

These diagrams reflect the results of experiment in which the occurrence of the irreversible component of the deformation process was ruled out, i.e., the state of the rock was within the elastic region. However, although the behavior of all type of rock is generally elastic (the unloading curve returns to the coordinate origin), the loading trajectory does not coincide with the unloading trajectory, and

$$\int_0^{\varepsilon_{\max}} \sigma(\varepsilon)\, d\varepsilon + \int_{\varepsilon_{\max}}^0 \sigma(\varepsilon) d\varepsilon > 0, \qquad (1)$$

which indicates pulsed-energy absorption. Here σ and ε are the stress and the corresponding relative strain of the rock and $\varepsilon_{\max}$ is the maximum relative strain of the rock attained in the experiment.

The fact that the above feature of the behavior of rocks is typical only of dynamic processes is supported by a comparison of curves 1 and 2 in Fig. 1a with curves 1, 2, and 6 in Fig. 1b (filled points refer to loading

Institute of Geophysics of the National Academy of Sciences of Ukraine, Kiev 252054. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 38, No. 2, pp. 162–169, March–April, 1997. Original article submitted July 27, 1994; revision submitted January 19, 1996.

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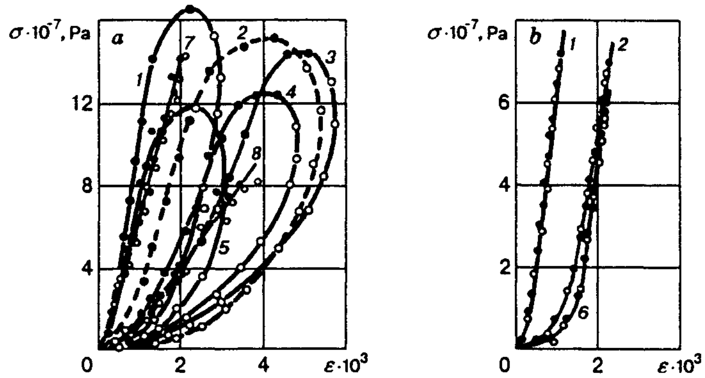

Fig. 1

TABLE 1

No.	Rock	Deposit	ρ, kg/m^3	n, %	v_{long}, km/sec	ν	$\sigma_0 \cdot 10^{-5}$, Pa
1	Pink quartzite	Ovruchskoye	2650	0.4	5.89	0.10	1980
2	Gray-green granite	Malinskoye	2600	1.0	3.42	0.22	1570
3	Gray-green limestone	Kamenets-Podol'skoye	2580	1.1	4.65	0.28	855
4	Alevrolite	Zapadno-Krestischenskoye	2580	5.4	3.26	0.19	520
5	Clay sandstone	Zapadno-Krestischenskoye	2360	11.0	2.53	0.17	—
6	Gray granite-gneiss	Ukrainian Shield	2610	0.8	4.29	0.21	1724

and open points to unloading). The curves were obtained on the same test complex for the same rocks (see Table 1) at a rate of increase in pressure of $3 \cdot 10^4$ Pa/sec. Note that, in static tests, a barely noticeable divergence of the loading and unloading branches of curves 2 and 6 was observed as the stress decreased below 4–4.5 MPa, which corresponds to 5.7–7.8% of the loading amplitude.

The shape of the dynamic-strain diagrams of rocks is not influenced by the errors of operation of the measuring complex, whose accuracy was checked by tests on dynamic compression of various model materials: metals (D-16T duralumin and cast lead) and spindle oil. These materials were chosen because they are homogeneous and do not have a distinct granular structure, and because their loading characteristics are independent of the time of loading.

Some results of the test experiments are given in [4]. Figure 1a shows dynamic $\sigma(\varepsilon)$ diagrams for D-16T duralumin and spindle oil (curves 7 and 8) obtained in the test experiments (dynamic loading of the oil was performed by the scheme $\sigma_1 = \sigma_2 = \sigma_3 = p$ and $\varepsilon_1 > \varepsilon_2 = \varepsilon_3 = 0$; the filled points in curves 7 and 8 refer to loading and the open points refer to unloading). Analysis of curves 7 and 8 shows that, first, the dynamic values of the Young modulus for duralumin and the compressibility of the oil agree satisfactorily with the rated (static) values of these model materials, i.e., the effect of loading conditions on the deformation properties of the materials is insignificant in the chosen experimental procedure, and, second, their $\sigma(\varepsilon)$ diagrams do not have the shape of a loop typical of rocks.

From Fig. 1 and relation (1) one can conclude that, under dynamic loading, rocks are active dissipating media even in the elastic region. Therefore, deformation processes in rocks under loads that do not exceed the elastic limit will be called quasi-elastic.

Note that for rocks, as for other solids, the notion of the elastic limit is rather conditional, since the possibility of determining irreversible strain completely depends on the resolution of the measuring instrument. As early as 1911, T. Kármán [5] showed that for rocks it is impossible to define the elastic limit as the maximum stress at and below which the total strain is completely reversible. However, because of the necessity of specifying the limits of application of the laws of elasticity for engineering design, the notion of the conditional yield point, at which significant residual deformation occurs, was introduced. In modern design for structural materials, 0.2% strain is regarded as such a critical strain (Hodgkinson criterion), although other criteria are also known (for example, the Wertheim criterion, 0.05%). It is important that the uncertainty in the notion of significant residual strain is responsible for the uncertainty in the notion (and, hence, in the qualitative definition) of the elastic limit. Therefore, in planning and performing the experiments described below and in processing their results, we defined the region of quasi-elastic behavior of rocks from the following considerations.

It is known that the occurrence of irreversible deformation in solids is associated with the existence of structural crystal imperfection, owing to which the actual strength of the body and the boundary of irreversibility of a deformation process is significantly (by more than two orders of magnitude) smaller than the theoretical maximum of elastic strain [6]. The deformation of bodies is predominantly of an elastic character as long as structural imperfections of the rock (crystal) such as Schottky and Frenkel defects, dislocations, etc., which are inherent in the medium in the natural state, are in a state of relative equilibrium.

The dislocation motion occurring under loading is irreversible and is responsible for the occurrence of an irreversible strain component. Hence, it can be concluded that the elastic limit in terms of loading is the pressure at which the motion of structural imperfections of crystals begin, and the elastic limit in terms of strain is the corresponding strain of the body. Because of the great number and diversity of natural defects in rocks, direct quantitative calculations of the elastic limit are impossible in practice.

The start of dislocations is usually determined by the Peierls–Nabarro relation [6]

$$\tau = \frac{2G}{q} \exp\Big(-\frac{2\pi a}{qb}\Big). \tag{2}$$

Here τ is the tangential stress, G is the shear modulus, q is a coefficient that depends on the type of dislocation, and a and b are the crystal-lattice parameters.

From (2) it follows that dislocation motion is rapid with periodic weakening and restoration of bonds in the crystal lattice. Therefore, the stress τ is considerably smaller than the theoretical stress in an ideal lattice, which is $\tau_{\text{theor}} = G/2\pi$, according to Ya. I. Frenkel. For solids with a nearly linear relation $\sigma(\varepsilon)$, such as the majority of dense hard intrusive rocks (the subjects of this investigation), we can write

$$\tau_{\text{theor}}/\tau = \varepsilon_{\text{theor}}/\varepsilon_{\text{el}}, \tag{3}$$

where $\varepsilon_{\text{theor}}$ is the maximum theoretical elastic strain and ε_{el} is the elastic strain that corresponds to the actual elastic limit. From (2) and (3) we have

$$\varepsilon_{\text{el}} = \frac{4\pi\varepsilon_{\text{theor}}}{q} \exp\Big(-\frac{2\pi a}{qb}\Big). \tag{4}$$

Calculations by (4) show that the elastic limit in terms of strain ε_{el} depends on the system of rock-forming materials and changes within (4–16) $\cdot\ 10^{-2}$% for cubic system, (2.88–10.7) $\cdot\ 10^{-3}$% for triclinic, (5.06–18.9) $\cdot\ 10^{-3}$% for monoclinic, (2.93–10.9) $\cdot\ 10^{-3}$% for tetragonal, (2.12–7.89) $\cdot\ 10^{-3}$% for rhombic, and (2.07–7.71) $\cdot\ 10^{-7}$% for trigonal systems. Evidently, the critical value ε_{el} for rocks coincide most closely with the Wertheim criterion proposed as early as 1844, and it is an order of magnitude smaller than the Hodgkinson criterion, which is generally adopted for structural materials. With allowance for the fact that the irreversible deformation of a rock as a polymineral formation occurs together with irreversible deformation of the weakest rock-forming mineral, the above approach to finding ε_{el} was used to determine the elastic limit of some intrusive rocks.

The calculation results are given in Table 2, where $\dot{\sigma}$ is the loading rate, E is the Young's modulus of a rock at the corresponding loading rate, and σ_{el} is the elastic limit. Since the stress–strain relation for rocks depends on loading conditions (velocity), the elastic limit also shows a similar dependence [4]. For the intrusive rocks of the Ukrainian shield, this dependence is shown in Fig. 2 [29 experiments with 11 varieties of rocks were performed (see Table 2)], where the points correspond to the average values for each variety.

Figure 3 shows the specific energy of the deformation process per unit of the applied load versus the

TABLE 2

Rock	Deposit	$\dot{\sigma}$, GPa/sec	E, GPa	σ_{elast}, MPa
Gray-green granite	Malinskoye	278.8	132	348
		536.7	111	492
		350.0	124	460
Porphyrous granite	Korninskoye	215.0	61	292
		198.6	106	308
Granodiorite	Korostyshevskoye	234.0	126	268
		425.0	139	480
		444.0	89	432
Pink granite	Emelyanovskoye	355.0	140	448
		421.0	120	382
		234.0	121	310

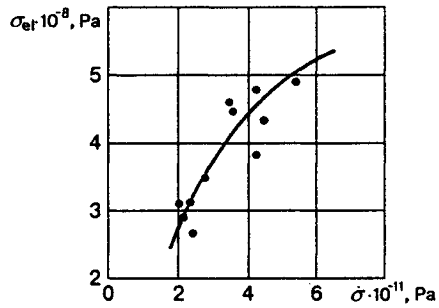

Fig. 2

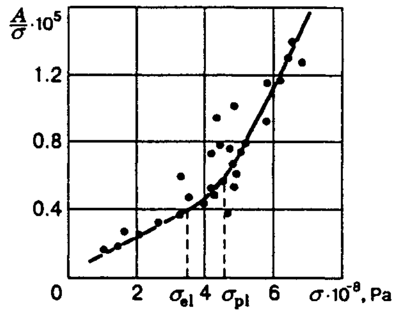

Fig. 3

loading amplitude. Note the presence of two distinct branches with different rates of increase in the energy capacity of rock deformation with an increase in the amplitude of dynamic loading. A comparison of stresses in the region of inflection with the data for σ_{el} in Table 2 shows that the left portion of the curve corresponds to the quasi-elastic state of the rock, and the right portion to the elastoplastic state. In Fig. 3, one can also distinguish the region of transition from the elastic σ_{el} to the plastic limit σ_{pl} at loading amplitudes of $3.5 \cdot 10^8$ to $4.6 \cdot 10^8$ Pa. The lower boundary of this range coincides with the calculated elastic limit given above.

According to Fig. 3, the relation between the energy capacity of the deformation process and loading intensity is parabolic for intrusive rocks:

$$A = 1.09 \cdot 10^{-14}\sigma^2, \qquad \text{J/m}^3. \tag{5}$$

Note that the general form of relation (5) indicates the linear behavior of intrusive rocks in the quasi-elastic region and confirms the possibility of using relation (3) for quantitative estimation.

Several mechanisms for the energy dissipation of dynamic loading (which does not give rise to residual strains) can be proposed which explain satisfactorily the shape of the strain diagrams for rocks in Fig. 1a. Lyakhov and Plyakova [7, 8] showed that, in terms of continuum mechanics, the noncoincidence of the loading and unloading branches at loads that do not exceed the elastic limit of the rock is apparently a consequence of the thermal dissipation of pulsed-loading energy due to the viscous properties of the deformed medium; the qualitative and quantitative differences between these branches are determined by the temporal configuration of the pressure pulse $\sigma(t)$, the dynamic-viscosity coefficient, and the static and dynamic elastic modulus of the medium.

The concept of a rock as an elastoviscous body in the quasi-elastic loading region explains satisfactorily a number of experimentally observed facts. First, as can be seen in Fig. 1a, under dynamic loading, the

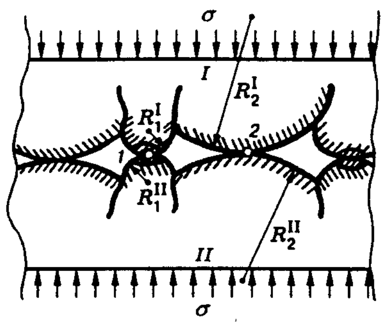

Fig. 4

amplitude values of strains lag behind those of stresses in phase [in the $\sigma(\varepsilon)$ diagrams, which shows up as a certain increase in the strains with a decrease in the acting stresses]. Second, Lyakhov [8] showed theoretically that, under loading by a bell-shaped pulse, viscous media should have qualitatively similar curves of $\sigma(\varepsilon)$, and this is supported by oscillograms of dynamic processes: the ratio of the time of increase to the time of decrease in the pulse pressure was 0.46–0.58 with an average value of 0.508. Third, proceeding from the model of a linear elastoviscous medium of [7] and taking into account the dependence of the dynamic-viscosity coefficient on the loading time [4], one can easily show that, under static loads ($t \to \infty$), such a medium undergoes linear elastic deformation, which is confirmed by the diagrams in Fig. 1b.

At the same time, in viscous media, deformation processes should proceed for some time after complete unloading of the medium from stresses. This was observed experimentally in compressible grounds [9, 10]. However, oscillograms of the experiments described above show that the duration of the signals $\sigma(t)$ and $\varepsilon(t)$ in hard rocks practically coincide [at least with accuracy to the following values of stresses, strains, and time, respectively: $(2.0\text{–}2.9) \cdot 10^5$ Pa, $(0.4\text{–}1.0) \cdot 10^{-5}$, and $(0.6\text{–}0.8) \cdot 10^{-4}$ msec]. In addition, proceeding only from the viscous mechanism of energy dissipation, one cannot explain the changes in the physical state of rocks caused by elastic dynamic loading, in particular, the decrease in the propagation velocity of longitudinal elastic waves [11] and the increase in filtration permeability [12] and mass-exchange properties [13]. These effects can be explained satisfactorily in terms of the mechanics of granular media, by visualizing a rock as a structurally nonuniform, deformable body.

We consider the deformation of a porous rock (Fig. 4) composed of mineral grains of various sizes which are in contact interaction with each other. We distinguish a characteristic element consisting of similar mineral formations I and II with identical mechanical properties. In this element, the distance between contact points 1 and 2 can be regarded as a characteristic pore size (microcrack). In static loading, the deformation process develops by intercrystalline (intergranular) displacement [14] with a relatively uniform load distribution over the mineral material of the rock. Since the loading level does not exceed the elastic limit, in unloading, deformation proceeds by the same mechanism, restoring the initial volume of the body.

In short-term dynamic loading, the displacement of the grains over the slippage surface has no time to develop, and the total load is taken by the contacts between the mineral grains. This feature of behavior of a granular medium is supported by the predominant damage of crystals at sufficiently high loading levels [14]. The qualitative aspect of the consequences of this interaction under loads that do not cause residual deformations of the medium can be analyzed on the basis of the following considerations.

Suppose that a characteristic element of a granular medium (Fig. 4) is subjected to dynamic loading. Taking into account that the grain size is, as a rule, much smaller than the length of waves excited in a rock specimen in laboratory tests [4] or in commercial types of dynamic loading (explosions, impacts, electric discharges, etc.), we shall assume that the interaction among individual mineral particles is of a quasi-static character. Hence, the contact stresses among the grains can be estimated using the Hertz problem [15]. Without disturbing the qualitative picture of the analysis, we assume that the curvature radii of the surfaces in contact at contact points are related by

$$R_1^{\rm I} = R_1^{\rm II} = R_1 < R_2^{\rm I} = R_2^{\rm II} = R_2, \quad R_2 = \alpha R_1.$$

Here the subscripts 1 and 2 refer to contact points 1 and 2, and the superscripts I and II to mineral formations I and II. In this schematic representation of the grain interaction, the relation between the maximum pressures at contact points 1 and 2 is $\alpha^{2/3}$. Thus, if the curvature radius of grains at point 2 is greater by a factor of two than that at point 1, the contact pressure at the latter point is higher by a factor of 1.6 than that at point 2, etc. Let the loading rate σ (Fig. 4) be such that, at a certain fixed loading level, the contact pressure between fairly coarse grains in contact does not exceed the elastic limit of the mineral material but approaches it. Then, from the foregoing and taking into account the different grain sizes of rocks, one can state that there are always contacts among grains (crystals) at which the contact stresses reach not only the elastic limit but also the strength limit. Owing to this, there is local damage of the mineral material to which the corresponding amount of dynamic-loading energy goes.

Upon unloading, the distance between the grains reaches the initial value, under the effect of elastic unloading, at points with a minor curvature, at which the pressures did not exceed $\sigma_{\rm elast}$. The deformation of the rock generally remains elastic. At points with a large curvature of the contact surface, the contact of the mineral material disappears, since the damaged and overcompacted [14] material partially fills the natural pore channels (Fig. 4), and a new pore channel appears which causes a change in some properties of the rock (for example, filtration permeability). Clearly, mineral grains with a large curvature of the contact surface are first to undergo local damage.

This mechanism of dynamic-loading energy dissipation agrees satisfactorily with microstructural and microtextural changes in rocks [14, 15], and explains some changes in the physical properties of rocks that are tested and used in technologies of intensification of borehole geotechnological processes [11–13, 16]. However, this mechanism cannot explain the above-mentioned features of dynamic-strain diagrams of rocks (in particular, the increase in strains behind the front of maximum stresses). Obviously, in real situations, both energy-dissipation mechanisms considered take place and supplement each other. Thus, it can be concluded that the features of the mechanical behavior of a rock as a deformable body under dynamic loading can be explained only in terms of continuum mechanics. To explain this behavior, and, moreover, to give it a theoretical basis, it is necessary to develop a unified approach in terms of the mechanics of continuous and discrete (granular, block, etc.) media.

In the elastoplastic loading region, the energy of the deformation process also varies as a parabolic law with an increase in the loading rate. The second branch of the curve in Fig. 3 is satisfactorily approximated by the relation

$$A = 3.94 \cdot 10^{-14}\sigma^2 - 1.198 \cdot 10^{-5}\sigma, \qquad {\rm J/m^3}. \tag{6}$$

A comparison of (5) and (6) shows that in the elastoplastic region, i.e., with the occurrence of irreversible strains, the rate of increase in the specific energy of the deformation process is greater by a factor of approximately 3.6 than that in the quasi-elastic region.

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